Comments on "Improved limit on quantum-spacetime modifications of Lorentz symmetry from observations of gamma-ray blazars"

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We address several criticisms by Amelino-Camelia of our recent analyses of two observational constraints on Lorentz violation at order E/M_{Planck}. In particular, we emphasize the role of effective field theory in our analysis of synchrotron radiation, and we strengthen the justification for the constraint coming from photon annihilation.

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I. INTRODUCTION

In a recent paper [1] we presented a new constraint on Lorentz violation of order E/M_{Planck} in the electron dispersion relation, based on the observation of synchrotron radiation from the Crab nebula. This constraint improved by a factor of one billion the previous best constraint, which was based on the observation of photon absorption from distant blazars [2, 3, 4] on the infrared background radiation. (The latter constraint limits the relevant Lorentz violating parameters to be of order unity in Planck units.) Both of these analyses were criticized in a recent paper by Amelino-Camelia Ref. [5]. The purpose of this note is to respond to these comments, clarifying our strategy and correcting some points that have been misconstrued in [5].

II. THE SYNCHROTRON CONSTRAINT

A. Theoretical Framework of Effective Field Theory

Ref. [5] observes that the synchrotron constraint of Ref. [1] relies on assumptions about the *dynamics* of electrons and electromagnetic fields rather than just the *kinematical* dispersion relations like some other constraints. It is stated there that if dynamical assumptions are made then "we are back to our starting point, we are actually proposing a full quantum-gravity theory, with all the uncertainties and risks of inconsistencies that plague quantum-gravity research." It is true that constraints derived using kinematics and dynamics are qualitatively different than constraints derived from kinematics alone. However, we do not agree that our assumption about low energy dynamics is equivalent to proposing a full quantum gravity theory.

The essential assumption in Ref. [1] about the full quantum gravity theory is that at energies low compared to the Planck energy electrons and the electromagnetic field can be described by an effective field theory preserving gauge invariance and rotation invariance. The only aspect of the interactions in this effective field theory that we use is that the modifications to the usual Lorentz-invariant QED equations of motion are suppressed by an inverse power of the Planck mass.

Let us elaborate a bit on how we view the significance of the effective field theory assumption. At low energies, physics is well described by the standard model and general relativity, which are both believed to be effective field theories. Since the particle energies even in astrophysical observations are far below the Planck energy it is reasonable to simply extend the low energy framework that we know is accurate. Ref. [5] comments that the dynamics we adopt "appear to be consistent with a classical and continuous spacetime, while most authors would expect deformed kinematics at the Planck scale to be the result of non-classical (discrete, noncommutative,...) aspects of spacetime structure, which should have equally dramatic (but presently unknown) consequences for dynamics." Exotic possibilities at the Planck scale do not preclude a low energy effective field theory description however. Some examples of this are lattice field theories, effective field theories of the low energy degrees of freedom of condensed matter systems [1],

[1] Ref. [5] states that the superluminal dispersion relations considered in Refs. [3, 4] are "conceptually disfavored" because ordinary media always lead to subluminal velocities. However, there exist condensed matter systems, like Bose-Einstein condensates (see e.g. [6]) or superfluid ^{3}He-A (see e.g. [7]), where Lorentz invariance emerges at low energies for quasiparticles while superluminal dispersion characterizes the high energy propagation.

and noncommutative field theory [8]. While we feel that the effective field theory assumption is quite reasonable, it is certainly conceivable that quantum gravity does not satisfy it. The constraints derived in Ref. [1] apply only modulo the effective field theory assumption.

The starting point for an analysis of possible quantum gravity effects (such as Lorentz symmetry violation) using effective field theory is to add to ordinary field theory the operators that realize the effects. One can then characterize the observational consequences. This approach has been extensively pursued in the case of renormalizable field theory [9, 10].

In the synchrotron case, the effective field theory being modified is QED. Since there are suggestions from quantum gravity that Lorentz symmetry might be violated with a suppression by $M_{\rm Planck}^{-1}$, we introduce in the low energy effective theory dimension five operators that violate Lorentz symmetry. We assume this is the only standard symmetry that is broken, so in particular we assume both rotation and gauge invariance are preserved. [2] The dispersion relations for free electrons and photons determine most of the terms in the effective field theory. In addition to momentum dependence, there can be polarization and chirality dependence of the dispersion. These possibilities correspond to different choices of the effective field theory. It was recently shown in [11] that there are only three additional terms quadratic in the fields that contribute to the dispersion relation $E^2(p)$ at cubic order in the momentum. In particular, the deformation parameters for left and right circular polarized photons must be negatives of each other, while the parameters for each electron chirality are independent. For the synchrotron constraint, the polarization dependence is irrelevant in the interesting region of parameter space. We assumed in our synchrotron calculation that the Lorentz violating parameters for electrons are chirality independent. It may be possible to lift this restriction and still get powerful constraints.

The interaction terms in the effective field theory arise from minimal coupling (replacement of ∂ by $\partial + ieA$) and non-minimal coupling in dimension five operators of the form $\bar{\psi}\Gamma^{\alpha\beta}\psi F_{\alpha\beta}$, where $\Gamma^{\alpha\beta}$ is built from gamma matrices and possibly a factor of the preferred timelike direction u^α. All terms other than the usual minimal coupling are suppressed by a factor of $1/M_{\rm Planck}$ hence can be neglected for our purposes, since the unsuppressed Lorentz invariant interaction terms dominate.

B. Objections to derivation of synchrotron constraint

There are three main theoretical objections in Ref. [5] to our derivation of synchrotron radiation in the presence of Lorentz violation. We respond to these objections below. First, however, we wish to address the criticism that the observation of Crab synchrotron emission is "at best a promising conjecture". We believe it is actually much more than a conjecture. The interpretation of the two observed humps in the emission spectrum from Crab and other supernova remnants as due to synchrotron and inverse Compton (IC) is the working hypothesis in the literature (see e.g. [12], [13]). This hypothesis is able to explain the observed fluxes with success at least up to 20 TeV, with a value of the magnetic field that is measured with consistent results by several methods. Uncertainties remain concerning the possible presence of more than one population of accelerated electrons and the specific mechanism responsible for the highest energy IC photons (e.g. synchrotron-self-Compton (SSC) plus IC on infrared and CMBR photons or hadronic contributions [12, 14]), but the synchrotron nature of the first hump is widely accepted. Thus we think that our adoption of the "standard model" for the Crab emission is well-justified. We now turn to the theoretical objections in Ref. [5] about the derivation of the synchrotron constraint.

1. Validity of heuristic formula for the cutoff frequency of synchrotron radiation

Ref. [5] states that we assume the Lorentz invariant expression (Eqn. (4) in [1]) for the synchrotron cutoff frequency $\omega_c(E)$ is valid in the Lorentz violating case. This is not an assumption—it is straightforward to verify explicitly and was done. As we state in our paper, this is a purely kinematical result. It involves the radius of curvature $R(E)$ of the electron trajectory, the angular width $\delta(E)$ of the synchrotron beam, and the group velocities of the electron and light, all of which are purely kinematical quantities (although the energy dependence of the first two depends on the dynamics).

[2] One could of course imagine that more symmetries are broken, but it would take a conspiracy for these to produce the usual symmetric physics when one broken symmetry alone would not. Hence we regard it as a fruitful strategy to begin by constraining (or looking for) minimal deviations from standard physics .

2. *Value of electron path radius of curvature $R(E)$*

Ref. [5] comments that we assume $R(E)$ is equal to its Lorentz invariant value, and that support for this assumption is given by proposing a new dynamics. We are not proposing a new, arbitrary dynamics, but merely implementing the dynamics given by the effective field theory approach discussed above. Since the modification of the electron field equation is strongly suppressed, $R(E)$ remains almost unchanged. More explicitly, gauge invariance determines the leading order interaction of the electrons and electromagnetic field via minimal coupling as described above. We show by calculation using this minimal coupling that while $R(E)$ is not equivalent to its Lorentz invariant value, it varies by some small relative amount which can be neglected in computing the cutoff synchrotron frequency.

3. *Value of the opening angle $\delta(E)$ of the synchrotron beam*

Ref. [5] states that the relation between the opening angle $\delta(E)$ and E is assumed to be unchanged. In fact it is not assumed, but rather argued (albeit briefly) that the scaling $\delta(E) \sim \gamma^{-1}(E)$ follows from the effective field theory. We expand on this argument here. For a fixed source term in the electromagnetic field equation, the effective field equation has a solution for the vector potential of the form $A = A_{LI} + A_{dev}$, where A_{LI} is the field that would be produced by the same source using the standard Maxwell equations. The deviation A_{dev} is a consequence of the Lorentz violation and contains a suppression factor of $1/M_{\rm Planck}$. Dimensional analysis indicates that it will thus be suppressed by a factor $\omega/M_{\rm Planck}$ where ω is the highest frequency in the problem. For the synchrotron emission from the Crab nebula, this frequency is roughly 1 GeV, leading to a suppression of A_{dev} by a factor of 10^{-19}. This is not competitive with the Lorentz invariant term, hence can be neglected. Thus the angular distribution of the radiation from a given source will be to a very good approximation the same as it is in the Lorentz invariant case.

In Ref. [5] evidence is given for large deviations of $\delta(E)$ by viewing the synchrotron process as an off-shell threshold phenomenon, and observing that the angular distribution in such a process can be very sensitive to Lorentz violation. However, it is a long way from such an observation for individual off-shell processes to a calculation of the classical, coherent effect of radiation from accelerating charges in a slowly varying magnetic field. It is clear from our field theoretic analysis that in the end the angle sensitivity discussed in Ref. [5] has no impact on the opening angle $\delta(E)$. Unlike in threshold phenomena, there is a Lorentz invariant zeroth order contribution that always dominates the synchrotron emission.

III. γ-RAY ABSORPTION WITH THE INFRARED BACKGROUND

We turn now to the weaker constraints that can be derived from the absorption of high energy gamma rays from blazars on the cosmic infrared background. Ref. [5] argues that the type of constraint derived in [2, 3, 4] is conditional on unverified assumptions about the source spectrum and IR background. We agree that there is some uncertainty here, although not as much as it is made out to be in [5]. Our reasoning was not very explicit in [4] however, so we elaborate here briefly on our viewpoint, taking the opportunity to strengthen the case somewhat.

Our starting point was the analysis carried out in reference [15]. There it was shown that using the most accurate model available for the infrared background the reconstructed spectrum of Mkn 501 shows no sign of anomalous pile up. Moreover, it was shown recently [16] that the SSC model accounts remarkably well for the intrinsic spectra of the blazars Mkn 501 and 421 (the latter in two different states of emission) consistently in both the synchrotron and IC regions, using the same IR background. Hence the statement in Ref. [2] that there is "no indication of Lorentz invariance breaking" up to 20 TeV is well justified.

The constraint in question corresponds to the statement that the soft photon threshold for absorption of a 20 TeV gamma ray should not be shifted upwards beyond 25 meV (50 μm), [3] the usual Lorentz invariant threshold ($\omega_{th} = m^2/k$) for a 10 TeV photon. To justify this statement in Ref. [4] we said that since there is no evidence for anomalies up to 10 TeV, it is unlikely that the threshold for 25 meV can be raised by more than a factor of order unity and remain consistent with the data. This was criticized in Ref. [5] on the grounds that one cannot observationally confirm absorption of any given *soft* photon energies, since it is only the effect on the spectrum of *hard* photons that is observed. We believe our justification of the constraint was inadequate, but the constraint is nevertheless justified

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[3] In Ref. [5] the author reported an error in our paper [4] saying that 25 meV photons correspond to a wavelength of 8 μm rather than 50 μm. This statement is not correct.

by the absence of anomalies in the reconstructed source spectrum out to 20 TeV rather than just out to 10 TeV. (This factor of two makes a significant difference because it appears cubed in the constraint.)

The grounds for such a constraint follow from the shape of the IR background spectrum reported in [15] If the absorption threshold for 20 TeV gamma rays were shifted up by a factor of two from 12.5 meV to 25 meV, that would eliminate all the absorption from the far infrared hump of the spectrum. This would lead to a sharp downturn in the reconstructed source spectrum above 10 TeV, which would be inconsistent with the SSC source model. (To be more precise about this effect it would be necessary to reconstruct the source spectrum allowing for Lorentz violation in the absorption on the IR background.)

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